Short Communications

IV Seminar on Categories and Applications

Bellaterra, 6 to 9 of June of 2007

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N-fold operads: braids, Young diagrams, and dendritic growth

Iterated monoidal categories are most famous for modeling loop-spaces via their nerves. There is still an open question about how faithful this modeling is. Examples of 2-fold monoidal categories include braided categories together with a choice of a very special braid to play the role of interchanger.

Examples of n-fold monoidal categories include ordered sets with n different binary operations. For each pair of operations an inequality expresses the interchange. We will present several example sets with their pairs of operations, beginning with max and plus on the natural numbers and proceeding to two new ways of adding and multiplying Young diagrams. The additions are vertical and horizontal stacking, and the multiplications are two ways of packing one Young diagram into another based respectively on stacking first horizontally and then vertically, and vice-versa.

N-fold monoidal categories generalize braided and symmetric categories while retaining precisely enough structure to support operads. The category of n-fold operads inherits the iterated monoidal structure. We will look at sequences that are minimal operads in the totally ordered categories just introduced, and discuss how these sequences grow. It turns out that the later terms are completely determined by the choice of initial terms, and if this choice is made carefully there appears a remarkable correspondence to certain natural processes. In fact, the growth rate of physical dendrites such as metallic crystals and snowflakes oscillates in a way directly comparable to that of our operads.

In relation to other topics at the conference, we will pose some open questions about how the nerves of n-fold operads might be described, and whether or not they do indeed form dendroidal sets. If time permits we will also discuss the possibility of using our families of n-dimensional Young diagrams to answer the open question of whether every n-fold loop space is represented by the nerve of an iterated monoidal category.

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