Publ. Mat. **69** (2025), 299–318
DOI: 10.5565/PUBLMAT6922502

THE PERIOD OF THE LIMIT CYCLE BIFURCATING FROM A PERSISTENT POLYCYCLE

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Abstract: We consider smooth families of planar polynomial vector fields $\{X_\mu\}_{\mu\in\Lambda}$, where Λ is an open subset of $\mathbb{R}^N$, for which there is a hyperbolic polycycle Γ that is persistent (i.e., such that none of the separatrix connections is broken along the family). It is well known that in this case the cyclicity of Γ at μ_0 is zero unless its graphic number $r(\mu_0)$ is equal to one. It is also well known that if $r(\mu_0)=1$ (and some generic conditions on the return map are verified), then the cyclicity of Γ at μ_0 is one, i.e., exactly one limit cycle bifurcates from Γ. In this paper we prove that this limit cycle approaches Γ exponentially fast and that its period goes to infinity as $1/|r(\mu)-1|$ when $\mu\to\mu_0$. Moreover, we prove that if those generic conditions are not satisfied, although the cyclicity may be exactly 1, the behavior of the period of the limit cycle is not determined.

2020 Mathematics Subject Classification: 34C05, 34C07, 34C23.

Key words: limit cycle, polycycle, cyclicity, period, asymptotic expansion, Dulac map.