Toric Topology and the Unstable Adams Novikov spectral Sequence

Given an $n$-dimensional, $q$-neighborly simple polyhedral complex $P$ one has the associated Borel space $B_T P$ and a moment angle complex $Z_P$. In the case that $P$ is a simple convex polytope there is a family of Quasi-Toric manifolds that sit over $P$. Recently, there has been much focus on the homotopy groups of these toric spaces. In this talk the relationship between the combinatorics of these structures and the methods from unstable homotopy theory will be discussed. Using co-simplicial objects one can work with co-simplicial resolutions over the category of unstable $G$ co-algebras over $BP_*(BP)$ to represent generators of the $R^1 PBP_*(B_T P)$ as certain star products in a particular unstable $G$ co-algebra. From this, the unstable co-action on $R^1 PBP_*(B_T P)$ can be computed. Hence, some input data for a particular Composite Functor Spectral Sequence that converges to the $E_2$ term of the Unstable Adams Novikov spectral Sequence can be computed. It turns out that through a range, the co-action on $R^1 PBP_*(B_T P)$ coincides with the co-action on a product of spheres whose dimensions depend on the combinatorics of $P$. As a result, the higher homotopy groups $B_T P$ and other toric spaces can be determined through a range that was previously unknown.

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