Introduction to Surface Group Representations and Higgs Bundles
Assignment 1

There is no formal submission of the assignments but you must work on them.

One of the students will present a solution and we will discuss alternatives.

Problem 1. We consider two charts on the he real line $\mathbb{R}$: the usual $(\mathbb{R}, \mathrm{Id}_{|\mathbb{R}})$ and the cubic chart $(\mathbb{R}, t \mapsto t^3)$.

- Are these charts compatible?

The maximal atlas containing the identity chart endows $\mathbb{R}$ with the structure of a differentiable manifold, say, the usual $\mathbb{R}$. On the other hand, the maximal atlas containing the cubic chart endows the same set $\mathbb{R}$ with the structure of a differentiable manifold, say, the cubic $\mathbb{R}$.

- Are the usual $\mathbb{R}$ and the cubic $\mathbb{R}$ the same differentiable structure?

We talked about differentiable functions $f : M \to \mathbb{R}$.

- Think about how you would define differentiability for a map $f : M \to N$ between two differentiable manifolds M, N.

We say that a differentiable map $f : M \to N$ is a diffeomorphism when it is bijective and its inverse is also a diffeomorphism.

- Are the usual $\mathbb{R}$ and the cubic $\mathbb{R}$ diffeomorphic?

Problem 2. We gave a differentiable structure to the circle S^1, by means of an atlas based on the angle parameterization. Use it to give an atlas for the cylinder $\mathrm{S}^1 \times (-1, 1)$ and the Möbius band.