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When looking at integrable structures (L involutive w.r.t. $[\cdot, \cdot]$): there are compact generalized complex manifolds that do not admit neither complex nor symplectic structures ($3\mathbb{C}P^2 \# 19\overline{\mathbb{C}P^2}$ by Gualtieri/Cavalcanti, and more by Rafael Torres, Wed 18:40).

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(are they knotted? are they linked?)

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We look at compact orientable 3-manifolds, up to $\mathrm{GDiff}^+(M)$:

- G_2^2-structures with $\rho_0 \neq 0$ always exist, are equivalent to $\rho_0 + \rho_3$ and are determined by the non-zero cohomology classes of (ρ_0, ρ_3).

G_2^2-structures

For a 3-manifold M, the "B_n-structure group" is $\mathrm{O}(4,3)$.
The real spin representation is 8-dim, with a $(4,4)$-pairing,
the non-null elements (non-pure) have stabilizer $G_2^2 \subset \mathrm{SO}(4,3)$.

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• G_2^2-structure with $\rho_0 = 0 \leftrightarrow M$ is the mapping torus of an orientable surface by an orientation-preserving diffeomorphism.

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• Cone of G_2^2-structures:

$$\{[\rho] \in H^\bullet(M, \mathbb{R}) \mid [\rho_0] \neq 0 \text{ and } [\rho_0][\rho_3] - [\rho_1][\rho_2] > 0\}$$
$$\bigcup \{(\alpha, \beta) \in C_1 \oplus H^2(M, R) \mid \alpha \cup \beta < 0\} \oplus H^3(M, \mathbb{R}),$$

where C_1 is the set of 1-cohomology classes with non-vanishing representatives (cf. Thurston)

B_3-Calabi Yau and G_2^2 structures

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which is the non-compact version of

$$\mathrm{SU}(3) \subset G_2 \subset \mathrm{SO}(7).$$

Obrigado!