Joint work with
Mario Garcia-Fernandez
and Carl Tipler.

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- (X, Ω) Calabi-Yau 3-fold: X complex with $\Omega \in \Omega^{3,0}_{hol}(X)$
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(Interest in Physics: equivalent to EM + SUSY + Bianchi in a Strominger compactification of the Heterotic String in the presence of NS fluxes.)

$\mathrm{SU}(3)$-holonomy, Strominger, and generalized Killing spinors

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This will take more than one slide...

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$$dH + \langle F \wedge F \rangle = 0.$$

This slide may hurt your sensibilities

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\begin{aligned}
[X + r + \xi, Y + t + \eta]_{\theta,H} = \\
&[X, Y] + L_X\eta - i_Y d\xi + i_Y i_X H \\
&- F(X, Y) + i_X dt - i_Y dr \\
&+ 2c(tdr) + 2c(i_X Ft) - 2c(i_Y Fr).
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A generalized metric on an exact Courant algebroid is actually equivalent to a usual metric g together with an isotropic splitting.

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so that a generalized admissible metric is equivalent to a usual metric g and an isotropic splitting of E, $E \cong T + \mathrm{ad}\, P + T^*$.

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But actually, we don't want only this one...

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$$\not{D}^\phi_- : S_+(V_-) \to S_-(V_-).$$

Finally, generalized Killing spinor equations

Given a generalized metric V_+, as before, and $\phi \in C^\infty(M)$, the *Killing spinor equations* for a spinor $\eta \in S_+(V_-)$ are given by

$$D_+^\phi \eta = 0,$$
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On a six-dimensinal spin-manifold

Theorem (Garcia-Fernandez,_____,Tipler)

Assume that E is exact. Then (V_+, ϕ, η) is a solution to the Killing spinor equations with $\eta \neq 0$ if and only if $H = 0$, ϕ is constant and g is a metric with holonomy contained in $SU(3)$.

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