Canonical metrics on holomorphic Courant algebroids

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Joint work with Garcia-Fernandez, Shahbazi, and Tipler, arXiv:1803.01873.

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$$\omega = g(J\cdot, \cdot) \in \Omega^{1,1}(X).$$

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We say that (X, J) is Kähler if J is integrable and there exists a hermitian metric $\omega \in \Omega^{1,1}$ that is closed, $d\omega = 0$. Kähler class: $[\omega] \in H^2(M, \mathbb{R})$.
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For integrable J, we have $d = \partial + \bar{\partial}$.

Calabi's conjecture (1954/1957)

THE SPACE OF KÄHLER METRICS

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Theorem 1. Given in M^n any real, closed, infinitely differentiable exterior form Σ of type $(1, 1)$ and cohomologous to $2\pi C^{(1)}$, there exists exactly one Kähler metric in Ω whose Ricci form equals Σ.

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On Kähler Manifolds with Vanishing Canonical Class

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PROPOSITION 1. *Let M_n be a compact, complex manifold admitting an infinitely differentiable Kähler metric with principal form ω and Ricci form Σ. If Σ' is any closed, real-valued, infinitely differentiable form of type $(1,1)$ and cohomologous to Σ, then there exists a unique Kähler metric with principal form ω' cohomologous to ω and Ricci form equal to Σ'. This metric is always infinitely differentiable; it is real analytic, if Σ' is analytic.*

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† There seems to be some question as to whether the interval in t for which one can solve for $\omega(t)$ is unbounded. This essential gap in the proof of Proposition 1 makes the results of this paper depend on the conjecture that a compact Kähler manifold admits a Kähler metric with any assigned, positive, differentiable volume element.

20 years later

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Calabi's conjecture and some new results in algebraic geometry

(Kähler manifold/Chern class/Ricci tensor/complex structure)

SHING-TUNG YAU

Theorem (Yau '77)

Let X be a compact Kähler manifold with smooth volume μ. Then there exists a unique Kähler metric with $\omega^n = n!\mu$ in any Kähler class.

Calabi-Yau metrics

For X admitting a holomorphic volume form Ω (Calabi-Yau manifold),

$$K_X := \Lambda^n T^* X \cong_\Omega \mathcal{O}_X,$$

we can use a multiple of $\Omega \wedge \overline{\Omega}$ as μ, say,

$$\omega^n = (-1)^{\frac{n(n-1)}{2}} i^n \Omega \wedge \overline{\Omega},$$

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Theorem (Yau '77)

Let (X, Ω) be a Calabi-Yau manifold. In each Kähler class there exists a unique Kähler metric with holonomy $SU(n)$.

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We say that a hermitian metric given by a form ω is:

- Kähler if $d\omega = 0$,
- pluriclosed or strong Kähler with torsion if $\partial\bar{\partial}\omega = 0$,
- balanced if $d\omega^{n-1} = 0$,
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But this does not relate to any cohomological quantity.

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Definition: An $SU(n)$-structure on X is a pair (Ψ, ω) such that

- $\omega \in \Omega^{1,1}(X)$ that is positive (i.e., g is riemannian),
- Ψ is a non-vanishing complex $(n, 0)$-form on X, normalized such that $\|\Psi\|_\omega = 1$ (that is, $\omega^n = (-1)^{\frac{n(n-1)}{2}} i^n \Psi \wedge \overline{\Psi}$)

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Definition: An $SU(n)$-structure (Ψ, ω) is a solution to the twisted Calabi-Yau system on X if:

$$(1)\ d\Psi - \theta_\omega \wedge \Psi = 0, \qquad (2)\ d\theta_\omega = 0, \qquad (3)\ \partial\bar{\partial}\omega = 0.$$

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- $(1) + (2) \Rightarrow$ the Bismut connection $\nabla^+ = \nabla^g - d^c\omega/2$ satisfies $hol(\nabla^+) \subset SU(n)$ (Calabi-Yau with torsion, recall $d^c = -J \circ d \circ J$).
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Moreover,

- the class $[\theta_\omega]$ is an invariant of the solutions: for fixed J, all solutions ω give the same class.
- when $[\theta_\omega] = 0 \in H^1(X, \mathbb{R})$, X admits a holomorphic volume form Ω and the equations are equivalent to the Calabi-Yau condition:

$$d\omega = 0, \qquad \omega^n = (-1)^{\frac{n(n-1)}{2}} i^n \Omega \wedge \overline{\Omega}.$$

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Proposition (Garcia-Fernandez–R–Shahbazi–Tipler)

A compact complex surface X admits a solution of the twisted Calabi-Yau system if and only if

- $X \cong K3$ or T^4, when $[\theta_\omega] = 0$,
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Observe: if X Hopf surface, then $H^2(X, \mathbb{R}) = 0$. What is the analogue of Kähler cone in Yau's Theorem?

Cohomologies in complex geometry

$$\begin{tikzcd}
 & H^{\bullet,\bullet}_{BC}(X) \arrow[dl] \arrow[d] \arrow[dr] & \\
H^{\bullet,\bullet}_{\bar\partial}(X) \arrow[dr] & H^{k}_{dR}(X,\mathbb{C}) \arrow[d] & H^{\bullet,\bullet}_{\partial}(X) \arrow[dl] \\
 & H^{\bullet,\bullet}_{A}(X) &
\end{tikzcd}$$

$$H^{p,q}_{BC}(X) = \frac{Ker\ d}{Im\ \partial\bar\partial}, \qquad H^{p,q}_{A}(X) = \frac{Ker\ \partial\bar\partial}{Im\ \partial \oplus \bar\partial}$$

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Observe: if X is a compact $\partial\bar{\partial}$-manifold (e.g. Kähler), all isomorphisms. Gauduchon, balanced and pluriclosed metrics give cohomology classes in, respectively, $H^{n-1,n-1}_{A}(X)$, $H^{n-1,n-1}_{BC}(X)$ and $H^{1,1}_{A}(X)$.

Theorem (Garcia-Fernandez–R–Shahbazi–Tipler)

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What about higher dimensions?

- There are many examples with no Aeppli classes. For instance, $\sharp_k(S^3 \times S^3)$ for any $k \geqslant 2$ (Clemens-Friedman). However, they have a large $H^3(X, \mathbb{R})$.
- “This” made us explore exact Courant algebroids.

Example of an exact Courant algebroid

Take $E = TX + T^*X$, with symmetric pairing $\langle X + \alpha, X + \alpha \rangle = i_X \alpha$, and, for some closed 3-form H, the bilinear (but not skew-symmetric) bracket

$$[X + \alpha, Y + \beta] = [X, Y] + L_X \beta - i_Y d\alpha + i_X i_Y H.$$

The bracket $[u, \cdot]$ is a derivation of both bracket and pairing, for $u, v, w \in \Gamma(TX + T^*X)$,

$$[u, [v, w]] = [[u, v], w] + [v, [u, w]],$$

$$\pi_{TX}(u)\langle v, w \rangle = \langle [u, v], w \rangle + \langle v, [u, w] \rangle,$$

and it satisfies

$$[X + \alpha, X + \alpha] = d i_X \alpha.$$

This kind of bracket is called a Dorfman bracket.

Biosketch of $TX + T^*X$

The graph of a 2-form ω and a skew bivector π are maximally isotropic subbundles of $TX + T^*X$. They are involutive with respect to the Dorfman bracket if and only if ω is presymplectic and π is Poisson. At the end of the 80's, Dirac structures were introduced as maximally isotropic involutive subbundles of $TX + T^*X$. They describe mechanical systems with symmetries and constraints.

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In 2003, generalized complex structures were introduced as orthogonal endomorphisms $\mathcal{J}$ of $TX + T^*X$ such that $\mathcal{J}^2 = -\,\mathrm{Id}$, and whose $+i$-eigenbundle is involutive. For J complex and ω symplectic structures,

$$\mathcal{J}_J = \begin{pmatrix} -J & 0 \\ 0 & J^* \end{pmatrix}, \qquad \mathcal{J}_\omega = \begin{pmatrix} 0 & -\omega^{-1} \\ \omega & 0 \end{pmatrix}$$

They interpolate between complex and symplectic structures and they are used in mirror symmetry. Generalized Kähler revived bihermitian geometry.

Definition: an exact Courant algebroid on a smooth manifold X consists of

- a vector bundle E fitting into the exact sequence
$$0 \to T^*X \to E \to TX \to 0$$
- a non-degenerate pairing $\langle \cdot, \cdot \rangle$ on E,
- a bilinear bracket $[\cdot, \cdot]$ on $\Gamma(E)$,

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Classification: any exact Courant algebroid E is isomorphic to $TX + T^*X$ for some $H \in \Omega^3_{cl}$. Actually $H^3(M, \mathbb{R}) = H^1(\Omega^2_{cl})$ classifies the isomorphism classes of exact Courant algebroids.

Holomorphic Courant algebroids

Definition: A holomorphic Courant algebroid Q is given by:

- a holomorphic sequence $0 \to T^*X \to Q \to TX \to 0$
- holomorphic metric $\langle \cdot, \cdot \rangle$ on Q,
- a Dorfman bracket $[\cdot, \cdot]$ on holomorphic sections.

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Classification (Gualtieri '10): isomorphism classes correspond to

$$H^1(\Omega^{2,0}_{cl}) = \frac{\operatorname{Ker} d \colon \Omega^{3,0} \oplus \Omega^{2,1} \to \Omega^{4,0} \oplus \Omega^{3,1} \oplus \Omega^{2,2}}{\operatorname{Im} d \colon \Omega^{2,0} \to \Omega^{3,0} \oplus \Omega^{2,1}}.$$

We have a map, rescaled by $2i$,

$$\partial\colon H_A^{1,1}(X) \to H^1(\Omega_{cl}^{2,0}).$$

We can talk about metrics and Aeppli classes compatible with Q:

- metric ω such that $[2i\partial\omega] = [Q]$.
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The map ∂ measures how far X is from being Kähler (the less Kähler, the less null).

- for a $\partial\bar{\partial}$-manifold (homologically Kähler), the map ∂ is identically zero, the Aeppli classes for any Q are just a copy of $H_A^{1,1}(X)$.
- for a Hopf surface $(\mathbb{C}^2 \setminus \{0\})/\mathbb{Z})$, the map ∂ is injective.

Definition: Let $H \in \Omega^{3,0} \oplus \Omega^{2,1}$ closed, defining an exact holomorphic Courant algebroid Q on X. An $SU(n)$-structure (Ψ, ω) is a solution of the twisted Calabi-Yau equation on Q if, for some $B \in \Omega^{2,0}$,

$$(1)\ d\Psi - \theta_\omega \wedge \Psi = 0, \qquad (2)\ d\theta_\omega = 0, \qquad (3)\ 2i\partial\omega = H + dB.$$

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Theorem (Garcia-Fernandez–R–Shahbazi–Tipler)

Let X be a compact complex surface endowed with an exact holomorphic Courant algebroid Q. If there exist a solution to the twisted Calabi-Yau system on Q, then there is exactly one solution in each Aeppli class in Σ_Q.

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Conjecture

Let X be a compact complex manifold endowed with an exact holomorphic Courant algebroid Q. If there exist a solution of the twisted Calabi-Yau system on Q, then there is exactly one solution in each Aeppli class in Σ_Q.

Perhaps someone else will in 20 years...

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Thank you for your attention!