Generalized Geometry, an introduction
Assignment 8

Weizmann Institute
Second Semester 2017-2018

There is no formal submission of the assignments but you are expected to work on them.

Problem 1. Prove that the following definitions for type of a linear generalized complex structure are equivalent.

- For an automorphism $\mathcal{J}$,

$$type(\mathcal{J}) = \frac{1}{2}\dim_{\mathbb{R}} V^* \cap \mathcal{J}V^*.$$

- For a subspace $L = L(E, \varepsilon)$,

$$type(L) = \dim_{\mathbb{C}} V_C - \dim_{\mathbb{C}} E.$$

- For a form $\varphi = \varphi_0 + \ldots + \varphi_n$,

$$type(\varphi) = \min\{k \mid \varphi_k \neq 0\},$$

  that is, the degree of the first non-vanishing component of φ.

Problem 2. Prove that for a linear generalized complex structure, the map $\pi_V \circ \mathcal{J}_{|V^*} : V^* \to V$ is a linear version of a Poisson structure.

Let J be a linear complex structure and $P \in \wedge^2 V$ be a linear Poisson structure.

- When is $\begin{pmatrix} J & P \\ 0 & -J^* \end{pmatrix}$ a linear generalized complex structure?
- What is its type?

Problem 3. We showed that a type m linear generalized complex structure is

$$\varphi = e^{B+i\omega} \wedge \Omega,$$

with $B, \omega \in \wedge^2 V^*$ and $\Omega = \theta_1 \wedge \ldots \wedge \theta_m \in \wedge^m V^*_{\mathbb{C}}$ such that $\Omega \wedge \overline{\Omega} \neq 0$.

- * Prove that there exist $B' \in \wedge^2 V^*$ such that $\varphi = e^{B'} \wedge \theta_1 \wedge \ldots \wedge \theta_m$.