# Generalized Geometry, an introduction
# Assignment 7

Weizmann Institute
Second Semester 2017-2018

There is no formal submission of the assignments but you are expected to work on them.

**Problem 1.** • Show that a maximally isotropic subspace $L \subset (V + V^*)_{\mathbb{C}}$ such that $L \cap \overline{L} = \{0\}$ can always be seen as the $+i$-eigenspace of a linear generalized complex structure.

**Problem 2.** Let $T$ be the reversing operator on $\wedge^{\bullet} V^*$ given, for $\alpha_j \in V^*$, by

$$(\alpha_1 \wedge \ldots \wedge \alpha_t)^T = \alpha_T \wedge \ldots \wedge \alpha_1.$$

We define a pairing $(\cdot, \cdot)$ on $\wedge^{\bullet} V^*$ with values on $\det V^* = \wedge^{top} V^*$ by

$$(\varphi, \psi) = (\varphi^T \wedge \psi)_{top},$$

where $\varphi, \psi \in \wedge^{\bullet} V^*$ and *top* denotes the top exterior power or component.

- For $v \in V + V^*$, prove $(v \cdot \varphi, \psi) = (\varphi, v \cdot \psi)$.
- For $x \in \mathrm{Cl}(V + V^*)$, prove $(x \cdot \varphi, \psi) = (\varphi, x^T \cdot \psi)$.
- For $g \in \mathrm{Spin}(V + V^*)$, $(g \cdot \varphi, g \cdot \psi) = \pm(\varphi, \psi)$.

**Problem 3.** Let $L = \mathrm{Ann}(\varphi)$ a maximally isotropic subspace

- Prove that $L \cap V = \{0\}$ if and only if $\varphi_{top} \neq 0$.
- Prove that $L \cap L(E', 0) = \{0\}$ if and only if $(\varphi, vol_{\mathrm{Ann}\, E'}) \neq 0$.
- For $L' = \mathrm{Ann}(\psi)$ be another maximally isotropic subspace, prove that $L \cap L' = \{0\}$ if and only if $(\varphi, \psi) \neq 0$.
- Do we need $L$ and $L'$ to be maximally isotropic subspace for the previous statement to be true?