Hyperbolic Workshop

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Using the mathematical software *The Geometer's Sketchpad*, we created the necessary tools to work with the Upper Half-Plane model of Hyperbolic Geometry in the same way that the Euclidian Geometry. That is, we made sketches to construct not only the basic objects as lines, segments or rays but circles with a given center and point, center and radius or the angle bisector as well as the parallelism angle, perpendicular line, horocycle,... and some of the triangle centers.

All the constructions were made using the environment that Sketchpad provides. That is, we construct the tools only using synthetic properties and the description of the lines and isometries in the Upper Half-Plane model. For instance, to construct a hyperbolic line that passes through two given points we use that this line is an euclidian circle perpendicular to the boundary line.

Once we had the basic tools implemented we began to use them by showing that the equivalent statements of the fifth Euclidian postulate are not fulfilled.

For example, using the tools it is possible to illustrate that in the Hyperbolic plane, given three different points, does not always exist a circumference meeting the three points. As the Sketchpad makes dynamic constructions we can plot three any points and then drag them. The circumference will be drawn only in the cases it exists so that we can find the existence conditions for the relation between the vertices.

As an example of application we made a second sketch that allow to work with triangles. Once we know how construct segments we can easily plot a hyperbolic triangle and then using the hyperbolic bisector, perpendicular bisector and perpendicular line tools to construct some of the remarkable points and lines.

We also made the isometries of the Upper Half-Plane and used the tools to construct tessellations. The isometries allow to see how the Euclidian and Hyperbolic length differ but the angles do not.

The tools are of public domain although the program *The Geometer Sketchpad* is not. They can be found in the web page of the *The Geometer Sketchpad*, http://www.keypress.com/sketchpad/general__resources/advanced__sketch__gallery/index.php, in the topic *Beyond Euclid* under the title *Half Plane Model of Hyperbolic Geometry* and also in my homepage (http://mat.uab.es/∼juditab/HypGeom.htm) where it is explained step by step how they are constructed.

These tools can be useful to study and see the differences between Euclidean and Hyperbolic Geometry in an easier and interactive way so that they are appropriated to be used in the beginning of the study of the Hyperbolic Geometry. These tools make a class more dynamic in the sense that the teacher can make constructions and demonstrations with the program in an easy and quick way without having to

draw again the picture to explain a similar but different case. Moreover, students can state hypothesis that the can be verified instantaneously in a visual way and, if necessary, make changes without having to clean the picture. However, they can not only be considered as mathematical education software applied in Geometry but they can be used in other areas as well specially in the Analysis area.

[1] Reventós, A., *Goemetria Axiomàtica*. Institut d'Estudis Catalans, Barcelona, 1993.

[2] Steketee, S., Jackiw, N., Chanan, S., *The Geometer Sketchpad. Dynamic Geometry Software for Exploring Mathematics. Reference Manual*. Key Curriculum Press, Emeryville, CA, 2001.