

## BANDLIMITED APPROXIMATIONS AND ESTIMATES FOR THE RIEMANN ZETA-FUNCTION

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**Abstract:** In this paper we provide explicit upper and lower bounds for the argument of the Riemann zeta-function and its antiderivatives in the critical strip under the assumption of the Riemann hypothesis. This extends the previously known bounds for these quantities on the critical line (and sharpens the error terms in such estimates). Our tools come not only from number theory, but also from Fourier analysis and approximation theory. An important element in our strategy is the ability to solve a Fourier optimization problem with constraints, namely, the problem of majorizing certain real-valued even functions by bandlimited functions, optimizing the  $L^1(\mathbb{R})$ -error. Deriving explicit formulae for the Fourier transforms of such optimal approximations plays a crucial role in our approach.

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**Key words:** Riemann zeta-function, Riemann hypothesis, argument, critical strip, Beurling–Selberg extremal problem, extremal functions, Gaussian subordination, exponential type.