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FIVE SOLVED PROBLEMS ON RADICALS OF ORE EXTENSIONS

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Abstract: We answer several open questions and establish new results concerning differential and skew polynomial ring extensions, with emphasis on radicals. In particular, we prove the following results.

If R is prime radical and δ is a derivation of R, then the differential polynomial ring $R[X;\delta]$ is locally nilpotent. This answers an open question posed in [**41**].

The nil radical of a differential polynomial ring $R[X;\delta]$ takes the form $I[X;\delta]$ for some ideal I of R, provided that the base field is infinite. This answers an open question posed in [**30**] for algebras over infinite fields.

If R is a graded algebra generated in degree 1 over a field of characteristic zero and δ is a grading preserving derivation on R, then the Jacobson radical of R is δ-stable. Examples are given to show the necessity of all conditions, thereby proving this result is sharp.

Skew polynomial rings with natural grading are locally nilpotent if and only if they are graded locally nilpotent.

The power series ring $R[[X;\sigma,\delta]]$ is well-defined whenever δ is a locally nilpotent σ-derivation; this answers a conjecture from [**13**], and opens up the possibility of generalizing many research directions studied thus far only when further restrictions are put on δ.

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Key words: Skew-polynomial extensions, differential polynomial rings, Jacobson radical, graded nil algebras.