

TANGENTS, RECTIFIABILITY, AND CORKSCREW DOMAINS

JONAS AZZAM

Abstract: In a recent paper, Csörnyei and Wilson prove that curves in Euclidean space of σ -finite length have tangents on a set of positive \mathcal{H}^1 -measure. They also show that a higher dimensional analogue of this result is not possible without some additional assumptions. In this note, we show that if $\Sigma \subseteq \mathbb{R}^{d+1}$ has the property that each ball centered on Σ contains two large balls in different components of Σ^c and Σ has σ -finite \mathcal{H}^d -measure, then it has d -dimensional tangent points in a set of positive \mathcal{H}^d -measure. As an application, we show that if the dimension of harmonic measure for an NTA domain in \mathbb{R}^{d+1} is less than d , then the boundary domain does not have σ -finite \mathcal{H}^d -measure.

We also give shorter proofs that Semmes surfaces are uniformly rectifiable and, if $\Omega \subseteq \mathbb{R}^{d+1}$ is an exterior corkscrew domain whose boundary has locally finite \mathcal{H}^d -measure, one can find a Lipschitz subdomain intersecting a large portion of the boundary.

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