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WEIGHTED SOLYANIK ESTIMATES FOR THE STRONG MAXIMAL FUNCTION

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Abstract: Let M_S denote the strong maximal operator on $\mathbb{R}^n$ and let w be a non-negative, locally integrable function. For $\alpha \in (0,1)$ we define the weighted Tauberian constant $\mathsf{C}_{\mathsf{S},w}$ associated with M_S by

$$\mathsf{C}_{\mathsf{S},w}(\alpha) := \sup_{\substack{E\subset\mathbb{R}^n \\ 0<w(E)<+\infty}} \frac{1}{w(E)} w(\{x \in \mathbb{R}^n : \mathsf{M}_\mathsf{S}(\mathbf{1}_E)(x) > \alpha\}).$$

We show that $\lim_{\alpha\to 1^-} \mathsf{C}_{\mathsf{S},w}(\alpha) = 1$ if and only if $w \in A_\infty^*$, that is if and only if w is a *strong Muckenhoupt weight*. This is quantified by the estimate $\mathsf{C}_{\mathsf{S},w}(\alpha) - 1 \lesssim_n (1-\alpha)^{(cn[w]_{A_\infty^*})^{-1}}$ as $\alpha \to 1^-$, where $c > 0$ is a numerical constant independent of n; this estimate is sharp in the sense that the exponent $1/(cn[w]_{A_\infty^*})$ can not be improved in terms of $[w]_{A_\infty^*}$. As corollaries, we obtain a sharp reverse Hölder inequality for strong Muckenhoupt weights in $\mathbb{R}^n$ as well as a quantitative imbedding of A_∞^* into A_p^*. We also consider the strong maximal operator on $\mathbb{R}^n$ associated with the weight w and denoted by M_S^w. In this case the corresponding Tauberian constant C_S^w is defined by

$$\mathsf{C}_\mathsf{S}^w(\alpha) := \sup_{\substack{E\subset\mathbb{R}^n \\ 0<w(E)<+\infty}} \frac{1}{w(E)} w(\{x \in \mathbb{R}^n : \mathsf{M}_\mathsf{S}^w(\mathbf{1}_E)(x) > \alpha\}).$$

We show that there exists some constant $c_{w,n} > 0$ depending only on w and the dimension n such that $\mathsf{C}_\mathsf{S}^w(\alpha) - 1 \lesssim_{w,n} (1-\alpha)^{c_{w,n}}$ as $\alpha \to 1^-$ whenever $w \in A_\infty^*$ is a strong Muckenhoupt weight.

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