STABILITY OF GENERALIZED LINEAR
WEINGARTEN HYPERSURFACES IMMERSED
IN THE EUCLIDEAN SPACE

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Abstract: Given a positive function $F$ defined on the unit Euclidean sphere and satisfying a suitable convexity condition, we consider, for hypersurfaces $M^n$ immersed in the Euclidean space $\mathbb{R}^{n+1}$, the so-called $k$-th anisotropic mean curvatures $H_k^F$, $0 \leq k \leq n$. For fixed $0 \leq r \leq s \leq n$, a hypersurface $M^n$ of $\mathbb{R}^{n+1}$ is said to be $(r, s, F)$-linear Weingarten when its $k$-th anisotropic mean curvatures $H_k^F$, $r \leq k \leq s$, are linearly related. In this setting, we establish the concept of stability concerning closed $(r, s, F)$-linear Weingarten hypersurfaces immersed in $\mathbb{R}^{n+1}$ and, afterwards, we prove that such a hypersurface is stable if, and only if, up to translations and homotheties, it is the Wulff shape of $F$. For $r = s$ and $F \equiv 1$, our results amount to the standard stability studied, for instance, by Alencar–do Carmo–Rosenberg [1].

2010 Mathematics Subject Classification: Primary: 53C42; Secondary: 53B25.

Key words: Euclidean space, Wulff shape, $k$-th anisotropic mean curvatures, $(r, s, F)$-linear Weingarten hypersurfaces, stable closed hypersurfaces.