

WEIGHTED SQUARE FUNCTION INEQUALITIES

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Abstract: For an integrable function f on $[0, 1)^d$, let $S(f)$ and Mf denote the corresponding dyadic square function and the dyadic maximal function of f , respectively. The paper contains the proofs of the following statements.

- (i) If w is a dyadic A_1 weight on $[0, 1)^d$, then

$$\|S(f)\|_{L^1(w)} \leq \sqrt{5}[w]_{A_1}^{1/2} \|Mf\|_{L^1(w)}.$$

The exponent $1/2$ is shown to be the best possible.

- (ii) For any $p > 1$, there are no constants c_p, α_p depending only on p such that for all dyadic A_p weights w on $[0, 1)^d$,

$$\|S(f)\|_{L^1(w)} \leq c_p [w]_{A_p}^{\alpha_p} \|Mf\|_{L^1(w)}.$$

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