COMPACTNESS OF HIGHER-ORDER SOBOLEV EMBEDDINGS

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Abstract: We study higher-order compact Sobolev embeddings on a domain $\Omega \subseteq \mathbb{R}^n$ endowed with a probability measure $\nu$ and satisfying certain isoperimetric inequality. Given $m \in \mathbb{N}$, we present a condition on a pair of rearrangement-invariant spaces $X(\Omega, \nu)$ and $Y(\Omega, \nu)$ which suffices to guarantee a compact embedding of the Sobolev space $V^m X(\Omega, \nu)$ into $Y(\Omega, \nu)$. The condition is given in terms of compactness of certain one-dimensional operator depending on the isoperimetric function of $(\Omega, \nu)$. We then apply this result to the characterization of higher-order compact Sobolev embeddings on concrete measure spaces, including John domains, Maz'ya classes of Euclidean domains and product probability spaces, whose standard example is the Gauss space.

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Key words: Compactness, Sobolev space, rearrangement-invariant space, isoperimetric function, almost-compact embedding, John domain, Maz'ya domain, product probability space, integral operator.