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POTENTIAL MAPS, HARDY SPACES, AND TENT SPACES ON SPECIAL LIPSCHITZ DOMAINS

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Abstract: Suppose that Ω is the open region in $\mathbb{R}^n$ above a Lipschitz graph and let d denote the exterior derivative on $\mathbb{R}^n$. We construct a convolution operator T which preserves support in $\overline{\Omega}$, is smoothing of order 1 on the homogeneous function spaces, and is a potential map in the sense that dT is the identity on spaces of exact forms with support in $\overline{\Omega}$. Thus if f is exact and supported in $\overline{\Omega}$, then there is a potential u, given by $u = Tf$, of optimal regularity and supported in $\overline{\Omega}$, such that $du = f$. This has implications for the regularity in homogeneous function spaces of the de Rham complex on Ω with or without boundary conditions. The operator T is used to obtain an atomic characterisation of Hardy spaces H^p of exact forms with support in $\overline{\Omega}$ when $n/(n+1) < p \leq 1$. This is done via an atomic decomposition of functions in the tent spaces $\mathcal{T}^p(\mathbb{R}^n \times \mathbb{R}^+)$ with support in a tent $T(\Omega)$ as a sum of atoms with support away from the boundary of Ω. This new decomposition of tent spaces is useful, even for scalar valued functions.

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