UNIFORMIZATION OF SMALL 3-ORBIFOLDS.

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Abstract We prove a uniformization theorem for small compact orientable 3-orbifolds, that implies Thurston's orbifold theorem.

Uniformisation des petites orbi-variétés de dimension trois

Resumé Nous démontrons un théorème d'uniformisation des petites orbi-variétés, compactes, orientables et de dimension trois. Il implique le théorème de Thurston de géométrisation des orbi-variétés compactes.

Version Française abrégée. Le résultat principal de cette note est un théorème d'uniformisation pour les petites orbi-variétés compactes, orientables et de dimension 3. Le redressement des actions lisses et non libres de groupes finis sur $\mathbb{S}^3$ est un cas particulier de ce résultat. En fait il implique le théorème de Thurston de géométrisation des orbi-variétés orientables de dimension 3. Thurston avait annoncé ce théorème de géométrisation en 1982, malheureusement il n'a pas publié sa preuve. D. Cooper, C. Hodgson et S. Kerckhoff viennent aussi d'obtenir une preuve du théorème de Thurston, différente de celle présentée ici.

Une orbi-variété compacte orientable est *petite* si les seules orbi-variétés de dimension 2, plongées de façon incompressible, sont des sphères avec 3 points de ramification et de caractéristique d'Euler ≤ 0, qui sont parallelles au bord.

Théorème 1 (Uniformisation des petites orbi-variétés). *Une petite orbi-variété de dimension 3, compacte, orientable, connexe et dont le lieu de ramification est non vide, est géométrique (i.e. admet une structure hyperbolique, Euclidienne, Sphérique ou fibrée de Seifert).*

Le corollaire suivant résout la conjecture de Smith généralisée [BM,chap.I]:

Corollaire 2. *Toute action lisse et non libre d'un groupe fini sur $\mathbb{S}^3$ qui préserve l'orientation est conjuguée a une action orthogonale.*

Toute orbi-variété de dimension 3, compacte, orientable, irreductible et atoroïdale s'obtient en recollant des orbi-variétés petites ou suffisamment grandes, le long de sphères avec 3 points de ramifications et de caractéristique d'Euler < 0. Le théorème 1 et le théorème d'hyperbolisation de Thurston, étendu aux orbi-variétés irréductibles, atoroïdales et suffisamment grandes, entrainent:

Corollaire 3 (Uniformisation des orbi-variétés). *Soit $\mathcal{O}$ une orbi-variété de dimension 3, compacte, orientable, connexe, irréductible et atoroïdale. Si son lieu de ramification est non vide, $\mathcal{O}$ est géométrique.*

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Introduction.

A 3-dimensional orientable orbifold is a metrizable space with coherent local models given by quotients of $\mathbb{R}^3$ by finite subgroups of $SO(3)$. For example, the quotient of a 3-manifold by a properly discontinuous group action naturally inherits a structure of a 3-orbifold. The set of points having a non-trivial *local isotropy group* is called the *ramification locus* of the orbifold. It is a codimension 2 trivalent graph. For a general background about orbifolds we refer to [BM].

A compact orientable 3-orbifold $\mathcal{O}$ is *irreducible* if it does not contain any bad 2-suborbifold (i.e. the union of two non-isomorphic discal 2-orbifolds along their boundaries) and if every orientable spherical 2-suborbifold bounds in $\mathcal{O}$ the quotient of the 3-ball by an orthogonal finite group action.

It is *small* if it is irreducible, its boundary is a (perhaps empty) collection of turnovers (i.e. 2-spheres with three branching points), and $\mathcal{O}$ does not contain any essential orientable 2-suborbifold.

The purpose of this note is to present a proof of a uniformization theorem for small 3-orbifold that implies Thurston's orbifold theorem.

Theorem 1 (Uniformization of small orbifolds). *A compact, orientable, connected, small* 3*-orbifold, with non-empty ramification locus, is geometric (i.e admits either a hyperbolic, Euclidean, spherical or Seifert fibered structure).*

A special case of Theorem 1 concerns non-free finite group actions on the 3-sphere $\mathbb{S}^3$ (cf. [BM]).

Corollary 2. *A smooth, orientation preserving, non-free, finite group action on* $\mathbb{S}^3$ *is conjugated to an orthogonal action.*

A compact orientable 3-orbifold is *topologically atoroidal* if any embedded toric 2-suborbifold is compressible or boundary-parallel. Any compact orientable irreducible and atoroidal 3-orbifold can be canonically split along a maximal (perhaps empty) collections of disjoint and pairwise non-parallel hyperbolic turnovers into either small or *Haken* 3-orbifolds (i.e. irreducible non-small 3-orbifolds in which every embedded turnover is compressible or boundary-parallel).

According to [BoP,Chap.VIII], Theorem 1 together with an extension of Thurston's hyperbolization Theorem to the case of Haken 3-orbifolds, imply :

Corollary 3 (Orbifold Theorem). *Let* $\mathcal{O}$ *be a compact, connected, orientable, irreducible* 3*-orbifold with non-empty ramification locus. If* $\mathcal{O}$ *is topologically atoroidal, then* $\mathcal{O}$ *is geometric.*

W. Thurston [Th1,2] had announced the Orbifold Theorem in 1982. He lectured about his proof, but unfortunatly he did not publish it. D. Cooper, C.Hodgson and S. Kerckhoff obtained recently another proof of this Theorem, different from the one presented in this note.

1. Deformation of hyperbolic cone 3-manifolds.

Let $\mathcal{O}$ be a compact connected orientable small 3-orbifold with non empty ramification locus Σ. We denote by $|\mathcal{O}|$ its underlying space. Its ramification locus Σ is a trivalent graph properly embedded in $|\mathcal{O}|$.

Let $\mathcal{N}(\Sigma^{(0)})$ be an open neighborhood of the vertices and $\Sigma^{(1)} = \Sigma - \Sigma^{(0)}$ the union of edges and circles of Σ. We consider the non compact manifold $M =$

$\mathcal{O} - \Sigma^{(1)} - \mathcal{N}(\Sigma^{(0)})$, with boundary a finite collection of three times punctured spheres.

Proposition 1.1. *Either the manifold M has a complete hyperbolic structure with finite volume and with totally geodesic boundary, or $\mathcal{O}$ admits a Seifert fibered or a I-bundle structure.* $\square$

From now on we assume that M is hyperbolic.

Definition 1.1. A cone 3-manifold *with topological type* $(|\mathcal{O}|, \Sigma)$ and curvature $k \in \mathbb{R}$ is a singular Riemannian metric on $|\mathcal{O}|$ with constant sectional curvature $k \in \mathbb{R}$ and cone-type singularities along Σ.

Each point of Σ has a neighborhood isometric to the metric cone with curvature $k \in \mathbb{R}$ on some singular orientable spherical cone 2-manifold.

A cone manifold of curvature $k = +1$, $k = 0$ or $k = -1$ is said to be *spherical*, *Euclidean* or *hyperbolic*, respectively. The *holonomy* representation of a hyperbolic, Euclidean or spherical cone 3-manifold is the holonomy of the non complete hyperbolic, Euclidean or spherical structure on the interior of M.

Theorem 1.1. *Either there exist a closed Euclidean cone 3-manifold with topological type $(|\mathcal{O}|, \Sigma)$ and cone angles strictly less than the orbifold angles of $\mathcal{O}$ or $\mathcal{O}$ has a hyperbolic, Euclidean or Seifert fibered structure.*

The proof of Theorem 1.1 involves the study of deformations of hyperbolic cone manifolds with topological type $(|\mathcal{O}|, \Sigma)$ and cone angles $\leq \pi$.

In the hyperbolic case we are interested in small cone angles. Hence we need to make the modifications (i) or (ii) below on the neighborhood of some vertices, according to the addition $\alpha + \beta + \gamma$ of the cone angles of the adjacent edges. The hyperbolic structure after modifications is still said to have topological type $(|\mathcal{O}|, \Sigma)$.

(i) when $\alpha + \beta + \gamma < 2\pi$, we remove an open neighborhood of v and we consider hyperbolic structures with totally geodesic boundary perpendicular to the singular edges;

(ii) when $\alpha + \beta + \gamma = 2\pi$, we remove the vertex v and we consider structures that have a cusp in the punctured neighborhood of v.

In the following, $C(t)$ denotes a hyperbolic cone 3-manifold with topological type $(|\mathcal{O}|, \Sigma)$ and cone angles $(2\pi t/m_1, \dots, 2\pi t/m_q)$, for $t \in [0,1]$, where $m_1, \dots, m_q$ are the ramification indices of the (ordered) edges and circles of Σ. With this notation, $C(0)$ corresponds to the complete hyperbolic structure of finite volume and geodesic boundary on M.

In order to study for which $t \in [0,1]$ $C(t)$ is hyperbolic, we consider an irreducibe component R_0 of the variety of representations

$R(M) := \mathrm{Hom}(\pi_1(M), SL_2(\mathbb{C}))$, that contains a lift of the holonomy ρ_0 of $C(0)$.

Definition 1.2. We define the subinterval $\mathcal{J}(\mathcal{O}) \subseteq [0,1]$ to be:

$$\mathcal{J}(\mathcal{O}) := \left\{ t \in [0,1] \;\middle|\; \begin{array}{l} \text{there exists a path } \gamma : [0,t] \to R_0 \text{ such that, for every } s \in [0,t], \\ \gamma(s) \text{ is a lift of the holonomy of a hyperbolic cone 3-manifold } C(s) \end{array} \right\}$$

Proposition 1.2. *$\mathcal{J}(\mathcal{O})$ is open.* $\square$

This a key step in the proof of Theorem 1.1. For a proof we refer to [BLP,§3] and [BoP,Prop.2.1 and Appendix B].

By hypothesis $0 \in \mathcal{J}(\mathcal{O})$, so $\mathcal{J}(\mathcal{O}) \neq \emptyset$. Let $(t_n)_{n\in\mathbb{N}}$ be an increasing sequence converging to $\sup \mathcal{J}(\mathcal{O})$, and let $\{C_n = C(t_n)\}_{n\in\mathbb{N}}$ be the corresponding sequence of hyperbolic cone 3-manifolds with topological type $(|\mathcal{O}|, \Sigma)$. We study the convergence of this sequence for the topology of the *geometric convergence* (i.e. the Gromov pointed bilipschitz convergence; cf. [BoP, Chap.III], [BLP,§3]).

Definition 1.3. The sequence $(C_n)_{n\in\mathbb{N}}$ collapses if $(\sup\{\mathrm{inj}(x) \mid x \in C_n\})_{n\in\mathbb{N}}$ goes to zero.

Proposition 1.3. *If the sequence $\{C_n = C(t_n)\}_{n\in\mathbb{N}}$ does not collapse, then $\mathcal{J}(\mathcal{O}) = [0,1]$ and the 3-orbifold $\mathcal{O}$ is hyperbolic.*

Proof. One shows that one can control the decay of injectivity radius in cone 3-manifolds with cone angles $\leq \pi$ and deduces from this a Compactness Theorem for cone 3-manifolds (cf. [BLP, Thm 3.1]). It follows that a pointed subsequence $(C_{n_k}, x_{n_k})_{n_k\in\mathbb{N}}$ converges geometrically to a pointed hyperbolic cone 3-manifold (C_∞, x_∞) with finite volume. According to [BLP,§6], one shows that C_∞ has topological type $(|\mathcal{O}|, \Sigma)$. Hence $\sup \mathcal{J}(\mathcal{O}) \in \mathcal{J}(\mathcal{O})$, and $\mathcal{J}(\mathcal{O}) = [0,1]$ since $\mathcal{J}(\mathcal{O})$ is open. $\square$

Proposition 1.4. *If the sequence $\{C_n = C(t_n)\}_{n\in\mathbb{N}}$ collapses, then one of the following occurs:*

(i) $\mathcal{J}(\mathcal{O}) = [0,1)$ and $\mathcal{O}$ is Euclidean, Seifert fibered or a twisted I-bundle over a closed non-orientable 2-orbifold.

(ii) $\sup \mathcal{J}(\mathcal{O}) < 1$ and there exist a closed Euclidean cone 3-manifold with topological type $(|\mathcal{O}|, \Sigma)$ and cone angles strictly less than the orbifold angles of $\mathcal{O}$.

Proof. If $\partial\mathcal{O} \neq \emptyset$, then in [BLP, §7] we prove that $\mathcal{O}$ is a twisted I-bundle over a non-orientable quotient of a turnover, by using an argument on the normal injectivity radius of the boundary turnovers and comparison techniques. In the cyclic case, the proof follows from [BoP]. The dihedral case reduces to the cyclic case by using that a closed orientable small 3-orbifold with dihedral singularities can be finitely covered by a cyclic one (cf. [BLP, §8]).

If there is a non-dihedral vertex, the proof follows from the Thick Vertex Lemma below. This lemma implies that $\mathrm{diam}(C_n) \to 0$ and that the rescaled sequence $(1/\mathrm{diam}(C_n)) \cdot C_n$ does not collapse. By taking a converging subsequence, we conclude that either $\mathcal{O}$ is Euclidean, when $\sup \mathcal{J}(\mathcal{O}) = 1$, or that assertion (ii) of the proposition holds, when $\sup \mathcal{J}(\mathcal{O}) < 1$. $\square$

Lemma 1.5 (Thick Vertex). [BLP,Lemma 3.4] *Let v be a singular vertex of a cone 3-manifold with cone angles $\leq \pi$ and of constant curvature in $[-1,1]$. If two of the edges adjacent to v have cone angles $< \pi - \delta$, then $\mathrm{inj}(v) > \varepsilon > 0$, where the constant $\varepsilon > 0$ depends only on $0 < \delta < \pi$.* $\square$

2. Euclidean cone structure on $\mathcal{O}$ with cone angles < orbifold angles.

Theorem 2.1. *Let $\mathcal{O}$ be a closed orientable connected small 3-orbifold. If there exists a closed Euclidean cone 3-manifold with topological type $(|\mathcal{O}|, \Sigma)$ and cone angles strictly less than the orbifold angles of $\mathcal{O}$, then $\mathcal{O}$ is a spherical 3-orbifold.*

If there is no vertex or if they are all dihedral, the proof of Theorem 2.1 uses Hamilton's Theorem [Ham]. We refer for this case to [BoP] and [BLP, §8]. From now on we assume that $\Sigma^{(0)} \neq \emptyset$.

Lemma 2.2. *$\mathcal{O}$ has finite fundamental group.*

Proof. An Euclidean cone structure on $\mathcal{O}$, with cone angles along Σ strictly less than the orbifold angles of $\mathcal{O}$, lifts to the universal cover $\widetilde{\mathcal{O}}$ to an Euclidean cone manifold structure with cone angles bounded away 2π along the singular set $\widetilde{\Sigma}$. By [BLP, Prop.4.1] $\widetilde{\Sigma}$ has finitely many components and vertices. Hence the fundamental group of $\mathcal{O}$ is finite since it acts isometrically on this Euclidean cone structure and $\widetilde{\Sigma}^{(0)} \neq \emptyset$ by hypothesis. □

The proof of Theorem 2.1 is by induction on the number of non-dihedral vertices in $\Sigma^{(0)}$. In the remaining of the proof, we fix a singular edge e of $\mathcal{O}$ with branching index $n_e > 2$ such that at least one of its adjacent vertices is non-dihedral.

Let $\mathcal{O}'$ be the closed orientable connected 3-orbifold obtained from $\mathcal{O}$ by relabelling the edge e with a branching index 2. It is small since it has the same topological type as $\mathcal{O}$. By induction hypothesis $\mathcal{O}'$ is spherical. The proof of Theorem 2.1 consists in deforming the spherical structure on $\mathcal{O}'$ to a spherical structure on $\mathcal{O}$. We deform the structure by deforming the holonomy representation of the non-complete spherical structure on $M = \mathcal{O} - \Sigma^{(1)} - \mathcal{N}(\Sigma^{(0)})$.

By choosing an orientation and a spin structure on M, we can lift holonomy representations to representations into $Spin(4) \cong SU(2) \times SU(2)$. The heart of the proof is the construction of a set $\mathcal{S}$ as follows:

Proposition 2.3. *There exists a semi-algebraic compact set $\mathcal{S}$ of conjugacy classes of representations of $\pi_1(M)$ into $SU(2) \times SU(2)$ such that:*

(i) Every point of $\mathcal{S}$ is the conjugacy class of the holonomy of a spherical cone manifold with topological type $(|\mathcal{O}|, \Sigma)$. The cone angle of this cone manifold at the edge e lies in $[2\pi/n_e, \pi]$ and the cone angles at the other edges are the same as the orbifold angles of $\mathcal{O}$ and $\mathcal{O}'$.

(ii) The subset $\mathcal{S}_0 \subset \mathcal{S}$ of points such that the cone angle at e lies in the open interval $(2\pi/n_e, \pi)$ is locally a real algebraic curve.

(iii) The set $\mathcal{S}$ is the closure of $\mathcal{S}_0$ and $\mathcal{S} - \mathcal{S}_0$ is finite.

(iv) The holonomy of $\mathcal{O}'$ belongs to $\mathcal{S}$ and it has a neighborhood homeomorphic to an interval $(\pi - \varepsilon, \pi]$ parametrized by the cone angle at e.

Proof of Theorem 2.1 from Proposition 2.3. By properties (ii) and (iii), $\mathcal{S}$ is topologically a compact graph. By Sullivan's local Euler characteristic theorem, property (ii) implies that the valency of each vertex in $\mathcal{S}_0$ is even. Thus $\mathcal{S}_0$ has an even number of ends. One of this ends is the holonomy of $\mathcal{O}'$ and $\mathcal{S}_0$ has at least another end which goes to some point $s \in \mathcal{S} - \mathcal{S}_0$. By property (iv), the point s is different from the holonomy of the initial spherical structure on $\mathcal{O}'$. The cone angle of the structure s at e is either π or $2\pi/n_e$. It cannot be π, otherwise it would contradict de Rham's rigidity theorem [Ro, Thm.4.3]. Thus the cone angle is $2\pi/n_e$, which proves that $\mathcal{O}$ is spherical. $\square$

3. Construction of $\mathcal{S}$ and of $\mathcal{S}_0$.

The algebraic quotient $X(M, SL(2,\mathbb{C})) = \mathrm{Hom}(\pi_1(M), SL(2,\mathbb{C}))//SL(2,\mathbb{C})$ is called the variety of characteres. It is an affine algebraic subset of $\mathbb{C}^N$. For every $\gamma \in \pi_1(M)$, $I_\gamma : X(M, SL_2(\mathbb{C})) \to \mathbb{C}$ denotes the algebraic function induced by the trace function.(cf. [CS])

The group $SU(2)$ acts on the variety of representations $\mathrm{Hom}(\pi_1(M), SU(2))$ by conjugation, and the topological quotient $X(M, SU(2)) = \mathrm{Hom}(\pi_1(M), SU(2))/SU(2)$ is a compact and real semi-algebraic subset of $V = X(M, SL(2,\mathbb{C})) \cap \mathbb{R}^N$. It is even locally algebraic at the points corresponding to nonabelian representations (cf. [BLP,§10].

Definition 3.1. A subset $\{\mu_1, \dots \mu_q\} \subset \pi_1(M)$ is a *meridian system* for Σ if each μ_i corresponds precisely to one meridian curve for each edge and circle of Σ.

Let $p_i : SU(2) \times SU(2) \to SU(2)$ denote the projection to the i-th factor, for $i = 1, 2$.

Theorem 3.1 (Local Rigidity). [BLP,Thm.10.5] *Let $\mathcal{O}$ be a spherical orbifold such that M is not Seifert fibered. If $\mu_1, \dots, \mu_q$ is a meridian system for Σ and $\phi : \pi_1(M) \to SU(2) \times SU(2)$ is a lift of the holonomy, then the map $(I_{\mu_1}, \dots, I_{\mu_q}) : X(M, SU(2)) \to \mathbb{R}^k$ is a local diffeomorphism at both conjugacy classes of $p_1 \circ \phi$ and $p_2 \circ \phi$.* $\square$

Proof of Proposition 2.5 from Theorem 3.1. Let $V = X(M, SL_2(\mathbb{C})) \cap \mathbb{R}^N$, we consider the algebraic set:

$$\begin{aligned} W = V \times V &\cap \{I_{\mu_i} \circ p_1 = I_{\mu_i} \circ p_2, \text{ for each meridian } \mu_i,\ i = 1, \dots, q\} \\ &\cap \{I_{\mu_i} \circ p_1 = I_{\mu_i} \circ p_1(\phi_0)), \text{ for each meridian } \mu_i \neq \mu_e\} \end{aligned}$$

where μ_e denotes the meridian of the edge e and ϕ_0 is the lift of the holonomy representation of the spherical orbifold $\mathcal{O}'$ obtained by relabelling the edge e with a branching index 2. The images of the meridians by representations in W are rotations with fixed rotation angles except for μ_e.

Let W_0 be the irreducible component of W that contains the holonomy of $\mathcal{O}'$. By the local rigidity Theorem 3.1 W_0 is a real curve. We take $\mathcal{S}$ to be the connected component of $W_0 \cap \{0 \leq \epsilon\, I_{\mu_e} \circ p_1 \leq 2\cos(\pi/n_e)\}$ that

contains the holonomy of $\mathcal{O}'$. It is a closed real semi-algebraic set. The sign of $\epsilon = \pm 1$ is uniquely determined by the choice of the spin structure on M.

Let $\mathcal{S}_0 = \mathcal{S} \cap \{0 < \epsilon\, I_{\mu_e} \circ p_1 < 2\cos(\pi/n_e)\}$. By definition, for representations in $\mathcal{S}$ the rotation angle of μ_e lies in $[2\pi/n_e, \pi]$, while for representations in $\mathcal{S}_0$ it lies in $(2\pi/n_e, \pi)$.

Properties (iii) and (iv) follow immediatly from the local rigidity theorem, which implies that $I_{\mu_e} \circ p_1$ is non constant on the curve W_0. Property (ii) follows from property (i) and the fact that the lift of the holonomy of a spherical cone structure with topological type $(|\mathcal{O}|, \Sigma)$ cannot be abelian. Hence we are left with the proof of property (i).

Let $A \subset \mathcal{S}$ be the subset of points that are the lift of the holonomy of a spherical cone structure with topological type $(|\mathcal{O}|, \Sigma)$. It is non-empty, because it contains the holonomy of $\mathcal{O}'$. It is open, because the set of non-abelian representations in $X(M, SU(2))$ is open in V ([BLP, Lemma 10.2]), and that deformations of holonomies imply deformations of structures (cf. [BLP, Prop.5.7]).

We prove that $A = \mathcal{S}$ by showing that A is closed. Let $\phi_n \in A$ be a sequence converging to $\phi_\infty \in \mathcal{S}$, we will show that $\phi_\infty \in A$. For $n \in \mathbb{N}$, let C_n be the spherical cone manifold with holonomy ϕ_n. The diameter of C_n is $\leq \pi$ by [BGP,Thm.3.6]. If the sequence $(C_n)_{n\in\mathbb{N}}$ does not collapse, then a subsequence converges geometrically to a spherical cone 3-manifold with topological type $(|\mathcal{O}|, \Sigma)$ and with holonomy ϕ_∞; whence $\phi_\infty \in A$.

We show now that the sequence $(C_n)_{n\in\mathbb{N}}$ cannot collapse.

When $(I_{\mu_e} \circ p_1)(\phi_\infty) = 0$, since the holonomy representations ϕ_n belong to a real algebraic curve, we may assume that they belong to a piecewise analytical path and that the function $I_{\mu_e} \circ p_1$ is eventually monotonic on this analytical path. This implies that the cone angle at the edge e increases along the path, because π is the supremum for this cone angle. By Schläfli's formula, the volume of C_n is also increasing. Since $\mathrm{diam}(C_n) \leq \pi$, the sequence $(C_n)_{n\in\mathbb{N}}$ cannot collapse.

When $(I_{\mu_e} \circ p_1)(\phi_\infty)) \neq 0$, since $\mathcal{O}$ has singularities which are not of dihedral type adjacent to e, each C_n has at least one vertex with two cone angles bounded uniformly away from π. The Thick Vertex Lemma 1.5 and the argument in the proof of Proposition 1.4 show the existence of a closed Euclidean cone 3-manifold with topological type $(|\mathcal{O}|, \Sigma)$ and with cone angles *bigger than or equal to* the orbifold angles of $\mathcal{O}$. This Euclidean cone manifold structure can be viewed as a CAT(0) structure on the orbifold. Therefore $\pi_1^o(\mathcal{O})$ is infinite by Haefliger's version of Cartan-Hadamard Theorem [Hae]. This contradicts Lemma 2.2. □

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