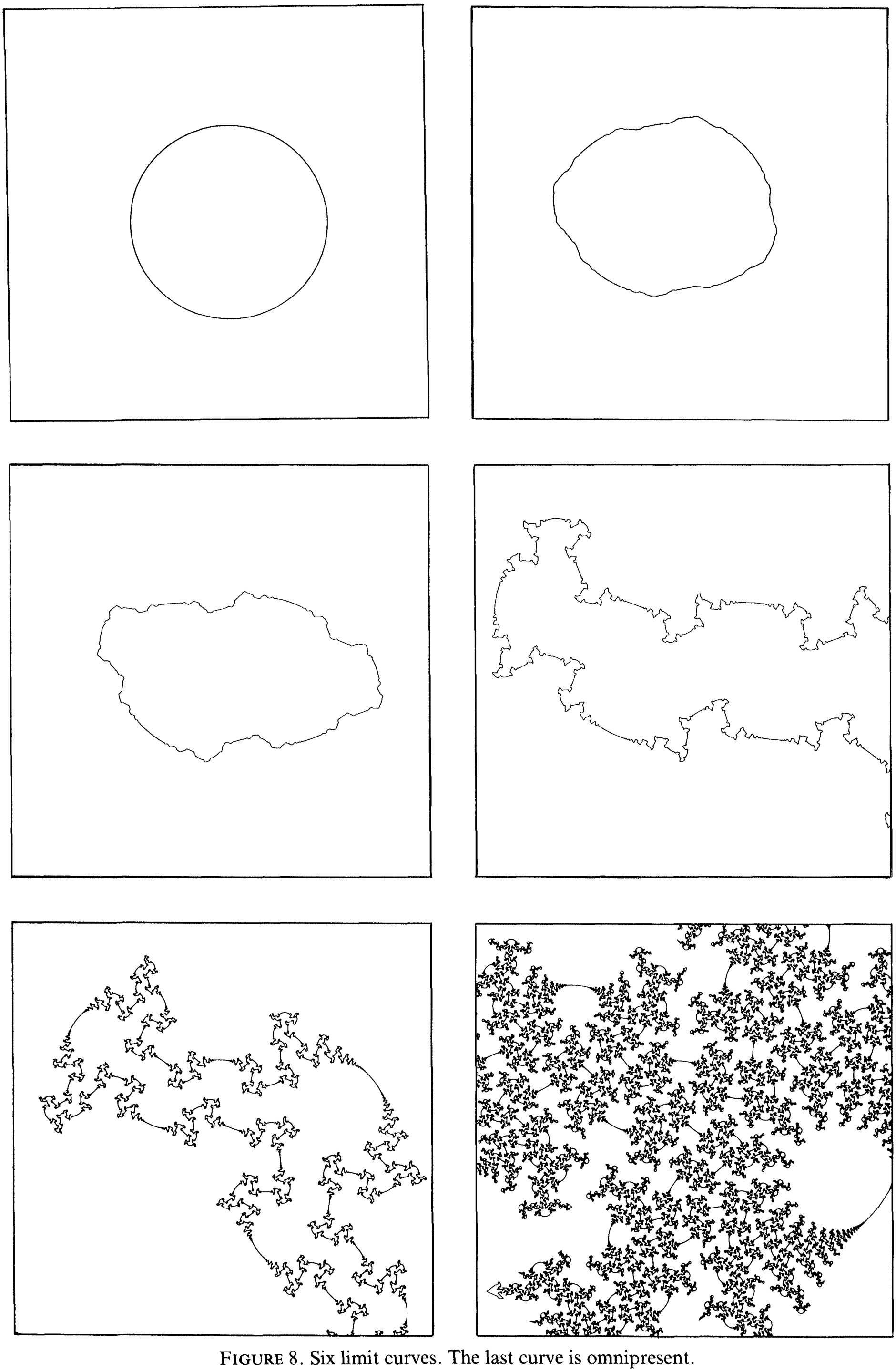

FIGURE 8. Six limit curves. The last curve is omnipresent.

Here are some computer sketches of examples in which the limit set is a curve. Poincaré was taken by the nondifferentiable nature of the curves which even have Hausdorff dimension > 1, as described in Sullivan's article [**Sul 1**].

The first limit set is a circle. This means the group leaves invariant a hyperbolic plane inside $\mathbf{H}^3$; in Poincaré upper half-space, hyperbolic planes appear as Euclidean hemispheres resting on **C**. The other groups are obtained by bending the original group. All the curves except the last are Jordan curves; the last curve actually fills **C**, but to make the computer actually fill **C** would require an amount of computer money (and foolishness) approximating the "defense" budget.

A group whose limit set is a circle is called the Fuchsian group (because of Poincaré's modesty), and groups whose limit sets are Jordan curves are called quasi-Fuchsian groups; they are obtained by quasi-conformal deformations of Fuchsian groups. The final group in our sequence of pictures cannot be obtained by a quasi-conformal deformation of a Fuchsian group, since the topology of the limit set has changed. How can we explain limiting phenomena such as this?

First we need to know something about the limiting geometry of hyperbolic surfaces, as the hyperbolic structure goes to infinity in Teichmüller space. One way that hyperbolic structures can go to infinity is that a curve, or system of curves, can be pinched:

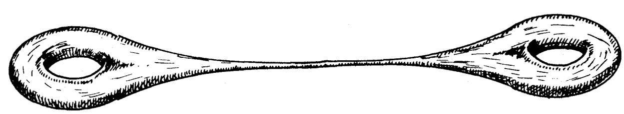

FIGURE 9. A surface with a pinched waist.

The hyperbolic metric develops a long, skinny waist to accomplish this.

To describe more generally how surfaces can go to infinity in Teichmuller space one needs a generalization of the notion of a simple closed curve.

A *geodesic lamination* λ of a hyperbolic surface S is a closed subset of S which is a disjoint union of complete geodesics on S (called the leaves of λ). A simple closed geodesic is one example of a geodesic lamination. To get other examples, consider a sequence of longer and longer simple closed geodesics. There is always a subsequence so that the pictures "converge", usually to an uncountable set of geodesics. A typical local cross-section of a geodesic lamination is a Cantor set, but other behavior can also occur. The 2-dimensional Lebesgue measure of a geodesic lamination is always 0.

A *transverse invariant measure*, μ, for a geodesic lamination can be thought of as a rule which assigns to each transverse arc α to λ a measure that is supported on $\lambda \cap \alpha$ and invariant under maps from one arc α to another arc β which take each point of intersection of α with a leaf of λ to a point of intersection of β with the same leaf. One can think of μ as a weighted counting of the leaves of λ, or as a measure of the amount of exertion required to cross a certain set of leaves. A lamination equipped with a transverse invariant measure of full support is a *measured lamination*. We also include here the trivial lamination consisting of no leaves and having 0 measure.