

# THE GROUP FIXED BY A FAMILY OF INJECTIVE ENDOMORPHISMS OF A FREE GROUP

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Errata and addenda (April 21, 2009)

<u>location</u>	<u>change</u>	<u>to</u>
page 8, line 18	$\beta'\gamma \approx \gamma\beta$	$\beta_1\gamma \approx \gamma\beta_2$
page 15, 5th last line	vertex. Here ... so	vertex, and thus
page 15, 4th last line	$\tilde{r}(Z) - \tilde{r}(Z - E_0) -  E_0 $	$\tilde{r}(Z) -  EZ  +  VZ $
page 17, line 32	immersion	covering
page 21, line 26	satisfies	satisfies
page 22, lines 14, 15, 25, 33	directed	downward directed
page 24, line 8	$\beta v$	$v\beta$
page 25, line 13	$u * F$	$x * F$
page 28, 3rd last line	$(x)\psi$	$\psi(x)$
page 29, lines 6–15	$v'$	$y$ (10 times)
page 29, lines 6–15	$x$	$z$ (3 times)
page 29, lines 6–15	$v$	$x$ (6 times)
page 29, line 26	is	if
page 29, line 26	such that $N' < N''$ .	such that $N' < N''$ , where a <i>submatrix</i> of $N$ is the square upper left corner of a conjugate of $N$ by some permutation matrix.
page 32, line 21	$Y_i$ -turn	$Y_{i-1}$ -turn
page 32, 10th last line	$[(e)\beta]_Z$	$[(e)\gamma]_Z$
page 40, line 11	$\beta$ -invariant	$\beta'$ -invariant
page 40, line 39	$N = \begin{pmatrix} A & b \\ c_1 + c_2 & r_1 + r_2 \end{pmatrix}$	$N = \begin{pmatrix} A & b_1 + b_2 \\ c & r_1 + r_2 \end{pmatrix}$
page 42, line 28	$Z'$	$Z$
page 43, line 12	$[\alpha][\beta][\alpha']$	$[\alpha'][\beta][\alpha]$
page 50, line 35	$((f)\beta^n)_i$	$((f)\beta^n)_i^{-1}$
page 51, line 11	If $z$ is	If $p$ is
page 54, 11th last line	$\text{PF}(\beta^{(4)}/Z_0^{(4)})$	$\text{PF}(\beta^{(5)}/Z_0^{(5)})$
page 58, line 9	$((z')\beta'^m)'$	$(z')\beta'^m$
page 62, lines 19, 35	$Z - EZ_0$	$EZ - EZ_0$
page 71, line 19	anser	answer
page 72, lines 12,13	new line	no new line
page 75, line 3	(preprint, 1995, 18 pages)	Trans. Amer. Math. Soc., <b>351</b> (1999), 1531–1550.
page 75, line 7	$\text{Out}(F_n)$ , (in preparation)	$\text{Out}(F_n)$ I. Dynamics of exponentially-growing automorphisms. Ann. of Math. <b>151</b> (2000), no. 2, 517–623.
page 75, line 17	(preprint, 1993, 6 pages)	Proc. Edinburgh Math. Soc. <b>41</b> (1998), no. 2, 325–332.
page 75, last two lines	(to appear)	<b>28</b> (1996), 255–263.

Additional reference containing a new proof of our main result, based on the theory of  $\mathbb{R}$ -trees:

Z. Sela, *The Nielsen-Thurston classification and automorphisms of a free group I*, Duke Math. J. **84** (1996), 379–397.

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Let  $\phi$  be an automorphism of a finitely generated free group  $F$ . On page 72, line 2, we ask if

$$\sum_{x * F \in ({}_1 F_\phi) / F} r(\text{Fix}(\phi x^{-1}))$$

is (finite and) bounded in terms of the rank of  $F$ . Armando Martino has pointed out to us that we are allowing multiple counting, in the sense that, for any  $f \in \text{Fix}(\phi x^{-1})$ , we also have  $f \in \text{Fix}(\phi(fx)^{-1})$ . Thus, the identity automorphism of the free group of rank one is an example where the sum is infinite, and answers our question in the negative.

To avoid multiple counting, one could construct the group  $\Phi = \langle S \mid R_1 \cup R_2 \rangle$  with generating set

$$S = \{(f, x) \mid x \in F, f \in \text{Fix}(\phi x^{-1})\}$$

and set of relators  $R_1 \cup R_2$  where

$$R_1 = \{(f, x)(g, x) = (fg, x), (f, x) = (y^{-1}fy, (y^{-1})\phi xy) \mid x, y \in F, f, g \in \text{Fix}(\phi x^{-1})\},$$

and

$$R_2 = \{(f, x) = (f, fx) \mid x \in F, f \in \text{Fix}(\phi x^{-1})\},$$

and ask if  $\Phi$  is finitely generated, and also ask if  $\Phi$  is free. Notice that

$$\langle S \mid R_1 \rangle \simeq \sum_{x * F \in ({}_1 F_\phi) / F}^* \text{Fix}(\phi x^{-1})$$

which is free, and its rank is the originally considered sum. Thus we are now adding the relations  $R_2$  to compensate for multiple counting.

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